Warm-up!

A) Evaluate $\sin\left(\frac{\pi}{6}+\frac{\pi}{3}\right)$

B) Evaluate $\cos\left(2\cdot\frac{\pi}{12}\right)$

C) Use common denominators to simplify the following into a single trigonometric function:

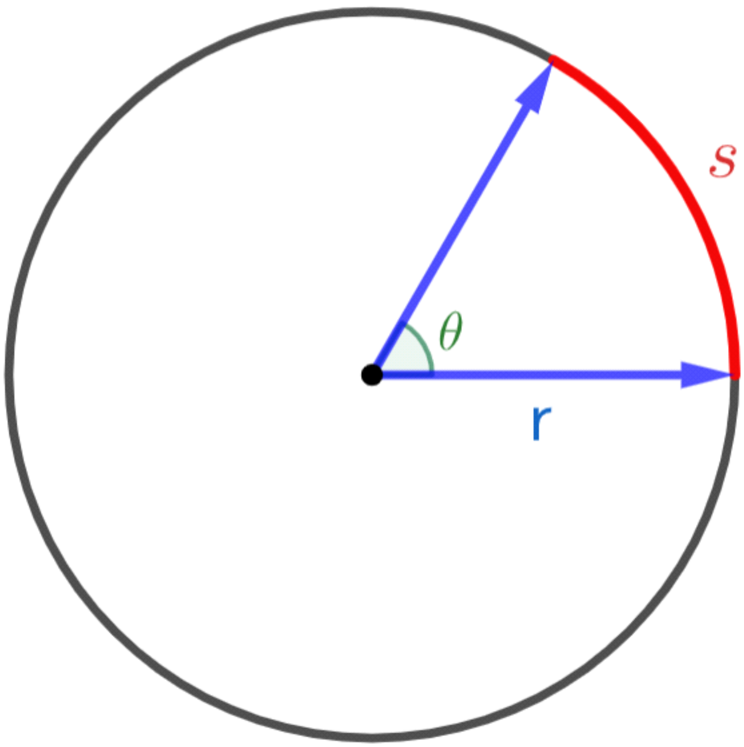
$$\frac{\csc\theta}{\cos\theta}-\frac{\cos\theta}{\sin\theta}$$

Advanced Trig Day 1:

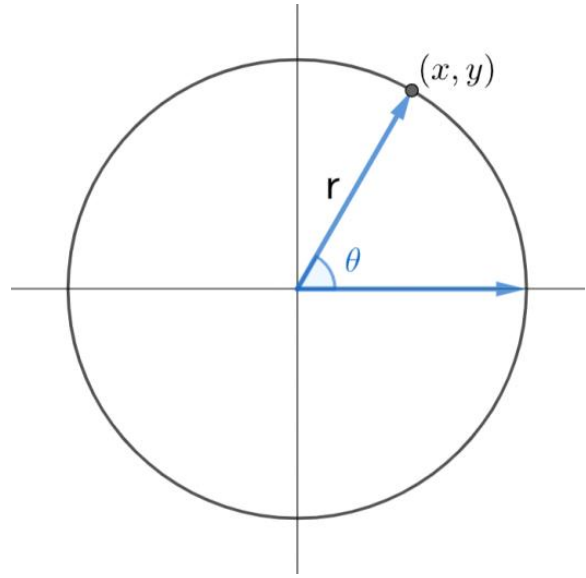
- Review a little Trig
- (3.12) Angle Sum/Difference Formulas
- (3.12) Double Angle Formulas

Homework: Yes

Review: $s = \theta r$



Review - Definition of Sine, Cosine, Tangent



Given an angle in standard position θ, a circle centered at the origin, and a point of intersection P…

- $\sin\theta$ is the ratio of the vertical displacement of P from the x-axis to the distance between the origin and P.
  $\sin\theta = \frac{y}{r} = \frac{y}{\sqrt{x^2+y^2}}$
- $\cos\theta$ is the ratio of the horizontal displacement of P from the y-axis to the distance between the origin and P.
  $\cos\theta = \frac{x}{r} = \frac{x}{\sqrt{x^2+y^2}}$
- $\tan\theta$ is the slope of the terminal ray.
  $$\tan\theta = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$$

Review - Trig Ratios

$$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

$$\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$$

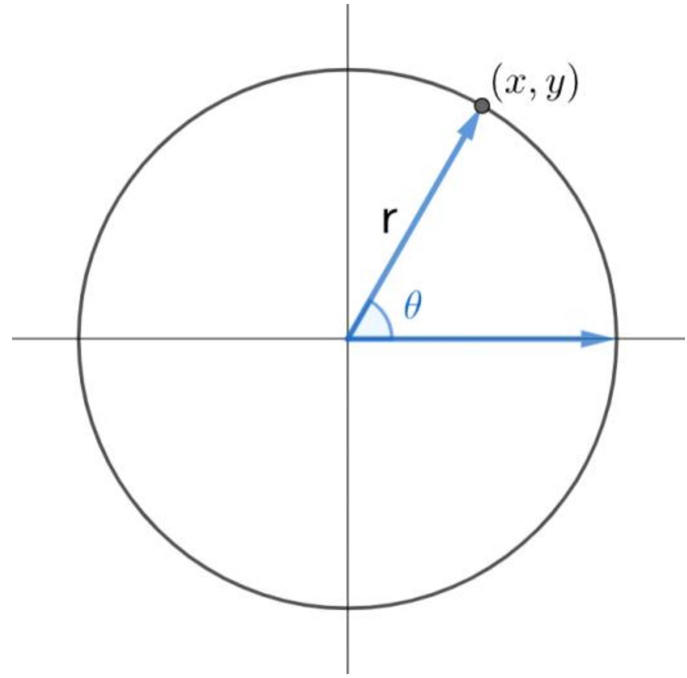
$$\tan\left(\frac{\pi}{4}\right) = 1$$

Review - Reciprocal Trig Functions

$\sec\theta =$

$\csc\theta =$

$\cot\theta =$



Review - Coterminal Angles and Reference Angles

$$\sin\left(\frac{11\pi}{6}\right) =$$

$$\cos\left(-\frac{7\pi}{4}\right) =$$

$$\sec\left(\frac{3\pi}{2}\right) =$$

Review - Pythagorean Identities

There are three. Do we remember them?

Angle Addition Formulas

$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

Examples:

$$\sin\left(\frac{17\pi}{12}\right) = \sin\left(\frac{3\pi}{4} + \frac{2\pi}{3}\right) =$$

You try

$$\cos\left(\frac{\pi}{3} + \frac{5\pi}{4}\right) =$$

Angle Subtraction Formulas

$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

$$\sin(\alpha - \beta) =$$

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$\cos(\alpha - \beta) =$$

Try to commit these enough to memory that you can try doing the next problems without looking back at the formula.

$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$
$$\sin(\alpha-\beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$
$$\cos(\alpha-\beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

Try to do these without looking at the formula

$$\sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) =$$

$$\cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right) =$$

$$\sin\left(\frac{\pi}{6} + \frac{7\pi}{4}\right) =$$

Double Angle Formulas

$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

$$\sin(\theta + \theta) =$$

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$\cos(\theta + \theta) =$$

Cosine Double Angle Formula - The Indecisive Identity

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$\cos(2\theta) =$$

$$\cos(2\theta) =$$

The Two/Six/Eight Formulas We Have Learned Today

$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$
$$\sin(\alpha-\beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$
$$\cos(\alpha-\beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \begin{cases} \cos^2\theta - \sin^2\theta \\ 2\cos^2\theta - 1 \\ 1 - 2\sin^2\theta \end{cases}$$

Simplify (Try to look at the formulas as little as possible to test how well you've already committed them to memory)

A) $\sin\left(\frac{\pi}{12}\right)\cos\left(\frac{\pi}{6}\right)+\cos\left(\frac{\pi}{12}\right)\sin\left(\frac{\pi}{6}\right)$

B) $\cos^2\left(\frac{3\pi}{8}\right)-\sin^2\left(\frac{3\pi}{8}\right)$

Simplify

C) $\sin\left(\frac{3\pi}{2}+\theta\right)$

D) $\frac{1-\cos(2\theta)}{1+\cos(2\theta)}$

Simplify - Example E

The function f is given by $f(x) = \frac{\cos x}{2 \sin x} - \frac{\sin x}{2 \cos x}$

Give the terms a common denominator. Then rewrite $f(x)$ as an expression in which cotangent is the only trigonometric function.

Simplify - Example F

The function g is given by $g(x) = \frac{\cot\theta - \tan\theta}{\cot\theta + \tan\theta}$

Rewrite $g(x)$ as an expression in which cosine is the only trigonometric function.

Example G

When graphed in standard position, the terminal ray of angle θ passes through the point $(-5,12)$.

Find $\cos\theta$

Find $\csc\theta$

Find $\cot\theta$

Example G

When graphed in standard position, the terminal ray of angle θ passes through the point $(-5,12)$.

Find $\sin(2\theta)$

Find $\cos\left(\frac{\pi}{2}+\theta\right)$

Find $\sin\left(\theta-\frac{\pi}{4}\right)$